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Final Report for AFOSR Contract AFOSR F-49620-92-0081
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W.H. Fleming's research focussed on risk-sensitive stochastic control problems on an infinite time horizon, and related questions in nonlinear H-infinity control theory. In risk-sensitive control, traditional expected integral of running cost criteria are replaced by expected exponential-of-integral of running cost criteria. On an infinite time horizon, the goal is to optimize the long term growth rate of such risk sensitive criteria. By introducing a small parameter which corresponds to a disturbance error intensity, dynamic games which arise in H-infinity control are obtained in the zero disturbance error limit. This technique provides a link between stochastic and deterministic approaches to the disturbance attenuation problem for control systems.

Publication 1 is concerned with infinite horizon, risk sensitive control of finite state machines. In this case, states, controls and disturbances all belong to finite sets. The finite state machine setting avoids purely technical issues which may obscure the main ideas in the development. Part I of Publication 1 concerns state feedback control, and Part II concerns output feedback (control with partial state information). For the output feedback case, the finite state machine H-infinity control problem was approached directly, without obtaining it as a zero disturbance error limit.

Publication 2 is concerned with a production planning system, in which machine breakdowns and repairs occur on a much faster time scale than the underlying planning scale. The zero disturbance error limit corresponds in this problem to a kind of averaging out of the fast time scale effects.

Reference 3 is a survey of recent results and open problems in risk sensitive control.

Risk sensitive control theory is related to problems of large deviations for Markov stochastic processes, and of eigenvalues and eigenfunctions of associated linear partial differential operators. Reference 4 is concerned with such problems, in cases when the Markov process is an ergodic diffusion which is nearly deterministic.

The following describes the recent work of Kushner. New and very practical convergence results for closed loop adaptive noise cancellation were obtained. Given the mean limit ODE for the stochastic approximation defining the adaptive algorithm for a closed loop ANC, we characterize the limit points. Under appropriate conditions, it is shown that as the dimension of the weight vector increases, the sequence of corresponding limit points converges in the sense of l_2 to the infinite dimensional optimal weight vector. Also, the limit point of

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the algorithm is nearly optimal if the dimension of the weight vector is large enough. The gradient of the mean square error with respect to the weight vector, evaluated at the limit, goes to zero in l_1 and l_2 as the dimension increases, as does the gradient with respect to the coefficients in the transfer function connecting the reference noise signal with the error output. Thus the algorithm is "nearly" a gradient descent algorithm and is indeed error reducing for large enough dimension. Under broad conditions, iterate averaging can be used to get a nearly optimal rate of convergence.

Much work was done on problems in modern high speed telecommunications. A large part of the aim was the demonstration of the usefulness of numerical methods of stochastic control theory for the design, analysis and control of multiplexing type systems and networks, as well as of the larger ATM type systems of which they form basic components. By exploiting the large size of the system (large number of users), the systems can be efficiently approximated by diffusion type processes, whether there is a control term or not, and for many types of control mechanisms. The basic controls are of the "low priority cell deletion" type, and various extensions. They might be state dependent, and we can obtain optimal controls for cost functions that weigh buffer overflow, controller cell deletion loss as well as queue length. The limit equations are an effective aggregation of the original system. They can be used for numerical as well as analytical work, and the numerical methods are based on them. Many forms of the problem have the same limit equations. The structure of the numerical approximations is close to that of the original problem, but is much simpler. It is shown that there are substantial savings in losses with the use of optimal control techniques. The optimal controls often have a surprisingly simple structure. The numerical methods can be used to get controls which appropriately balance the losses at the control with those due to buffer overflow. They can be used to get controls which minimize losses at the controller subject to constraints on buffer overflow. They can be used to explore various approximations, systems aggregations, the interaction of multiple source classes of different priorities. These are typical of many possibilities. The approach is a powerful useful tool for getting both qualitative and quantitative information on problems which would be hard to do otherwise.

Perhaps the major achievement was the completion of a book on Stochastic Approximation or recursive stochastic algorithms, for both constrained and unconstrained problems, no doubt the best and most comprehensive to date. The general motivation arises from the new challenges in applications that have arisen in recent years. There is a thorough treatment of both probability one and weak convergence methods for very general noise models. The convergence proofs are built around the powerful ODE (ordinary differential equations method), which characterizes the asymptotic points of the algorithm in terms of the asymptotic behavior of a "mean limit ODE" or analogous dynamical system. The method is particularly convenient for dealing with complex noise and dynamics, but also greatly simplifies the treatment of the more classical

cases. There is a thorough treatment of rate of convergence, iterate averaging, high dimensional problems, ergodic cost problems, stability methods for correlated noise, and decentralized and asynchronous algorithms. The basic ideas concerning the application of large deviations methods are discussed. The use of perturbed test function or perturbed state methods for complex correlated noise processes is explained. These methods are applicable generally to approximation and limit problems. The development proceeds from the simpler problems to the more complex, allowing the underlying ideas to be motivated and understood without excessive encumbrance. Many motivating examples are described, and convergence proofs for a wide range of problems (signal processing, learning theory, ergodic cost problems, noise cancellation, neural net training, etc) are given in order to illustrate the methods.

Publications

1. W.H. Fleming and D. Hernandez-Hernandez, Risk-sensitive control of finite state machines on an infinite horizon. Part I, to appear in SIAM J. on Control and Optimization. Part II, submitted to that journal.

2. W.H. Fleming and Q. Zhang, Risk-sensitive production planning of a stochastic manufacturing system, to appear in SIAM J. on Control and Optimization.

3. W.H. Fleming, Some results and problems in risk sensitive stochastic control, Comput. Appl. Math. vol 16 (1997) 99-115.

4. W.H. Fleming and S.-J. Sheu, Asymptotics for the principal eigenvalue and eigenfunction of a nearly first order operator with large potential, to appear in Annals of Probability.

5. A Note on Closed Loop Adaptive Noise Cancellation. To appear IEEE Trans on Automatic Control

6. Analysis of Controlled Multiplexing Systems via Numerical Stochastic Control Methods; IEEE Trans on Selected Areas in Communications. 1996

7. Stochastic Approximation Algorithms and Applications; Springer, 1997

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